

List Homomorphisms and Interval Digraphs

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Bordeaux Graph Workshop, November 19, 2010

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Joint with Arash Rafiey

currently at IDSIA, University of Lugano



Homomorphisms

Given digraphs G and H

A *homomorphism* $f : G \rightarrow H$ is a mapping $f : V(G) \rightarrow V(H)$ such that $xy \in E(G) \implies f(x)f(y) \in E(H)$

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Undirected graphs are viewed as symmetric digraphs

Homomorphism Problems

Given a fixed digraph H

Does an input digraph G admit a homomorphism to H ?

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G admits a 2-colouring if and only if it contains no odd cycle

CSP with fixed template H

H with $V(H)$ and relations $R_1(H), \dots, R_k(H)$

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H with $V(H)$ and relations $R_1(H), \dots, R_k(H)$

Does an input G admit a homomorphism to H ?

G has corresponding relations $R_1(G), \dots, R_k(G)$

Homomorphisms preserve all relations

Dichotomy Conjecture

Feder - Vardi, 1993, conjectured for any template H

The CSP problem for H is polynomial or is NP-complete

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Still open, very active

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The homomorphism problem for H is polynomial if H is bipartite or contains a loop; otherwise it is NP-complete

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If dichotomy holds for all digraph templates H then the dichotomy conjecture holds for all CSP

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List Homomorphism Problems

Given a fixed digraph H

Each vertex x of the input digraph G has a *list* $L(x) \subseteq V(H)$

List Homomorphism Problems

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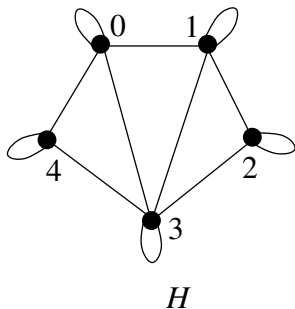
Each vertex x of the input digraph G has a *list* $L(x) \subseteq V(H)$

Is there a homomorphism $f : G \rightarrow H$ for which all $f(x) \in L(x)$?

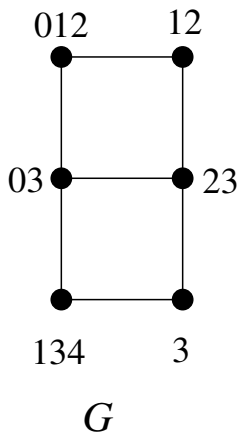
List Homomorphism Problems

Fixed graph H

Processors and connections



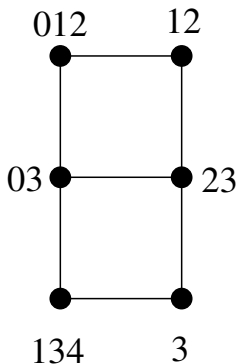
List Homomorphism Problems



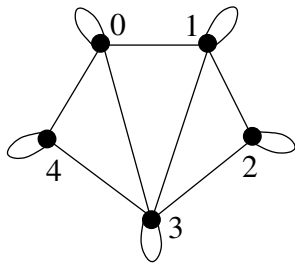
Input graph G

Tasks and communications

List Homomorphism Problems

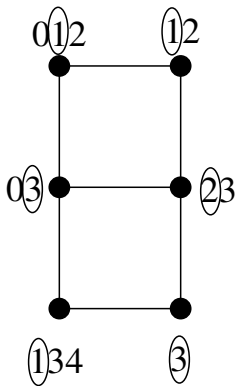


G

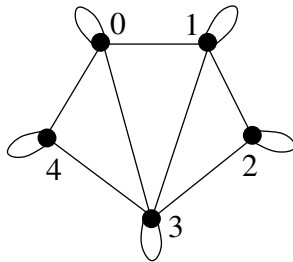


H

List Homomorphism Problems

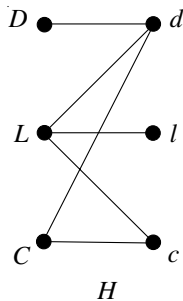
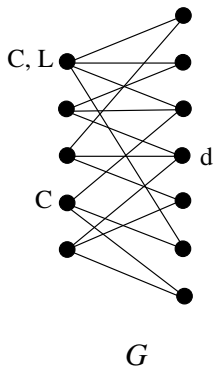


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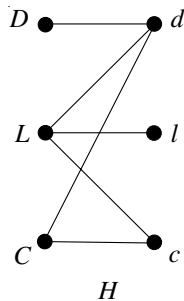
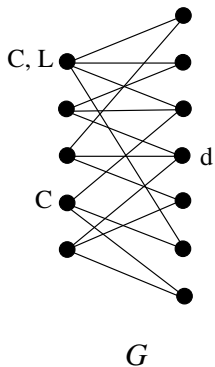


H

Another Example Application



Another Example Application



Lists

Lists of allowed decisions

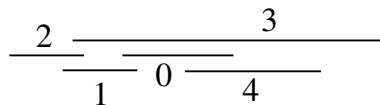


Reflexive Undirected Graphs

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Interval graph

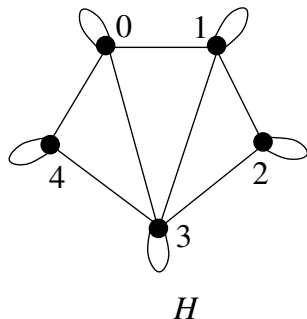
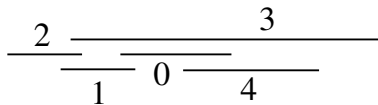
Vertices correspond to intervals and adjacency to intersection



Reflexive Undirected Graphs

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List Homomorphism Problem for Graphs

For a reflexive graph H

If H is an interval graph, then the problem for H is polynomial

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Feder+H 1998

Min Ordering

A min ordering $<$ of $V(H)$

$uv, u'v' \in E(H) \implies \min(u, u') \min(v, v') \in E(H)$

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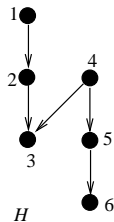
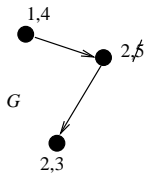


Theorem

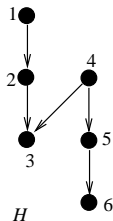
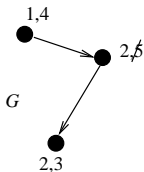
If H admits a min ordering, then the list homomorphism problem for H is polynomial

Gutjahr, Welzl, Woeginger 1992

Local Consistency Algorithm



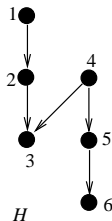
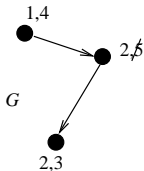
Local Consistency Algorithm



Min ordering

Each reflexive interval graph H admits a min ordering

Local Consistency Algorithm



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Reflexive interval graph H

The list homomorphism problem for H can be solved by a local consistency algorithm of width 1.

Interval Graphs

Lekkerkerker, Boland

H is an interval graph $\iff H$ has no AT and no hole

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AT = Asteroidal Triple

u, v, w , any pair joined by a path avoiding the neighbourhood of the third

Hole

Induced cycle of length at least four

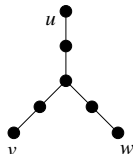
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For an irreflexive graph H

If H is a bipartite graph whose complement is a circular arc graph, then the problem for H is polynomial
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Feder+H+Huang 1999

Polymorphisms

A k -ary polymorphism on H

A homomorphism $\phi : H^k \rightarrow H$

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A polymorphism $\phi : H^k \rightarrow H$ with $\phi(u_1, \dots, u_k) \in \{u_1, \dots, u_k\}$

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Heredity property

The restriction of a conservative polymorphism is a polymorphism

Conservative Binary Polymorphisms

A conservative semi-lattice on H

A conservative binary polymorphism $f : H^2 \rightarrow H$ which is commutative and associative

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Min ordering

f is a conservative semi-lattice on H if and only if $f(u, v) = \min(u, v)$ for some linear ordering of $V(H)$

Conservative Ternary Polymorphisms

A conservative majority on H

A conservative ternary polymorphism $g : H^3 \rightarrow H$ such that

- $g(u, u, v) = g(u, v, u) = g(v, u, u) = u$

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Theorem

If H admits a conservative majority, then the list homomorphism problem for H is polynomial

Feder+Vardi 1993; Jeavons 1998

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Jeavons, Cohen, Gyssens 1997

Existence of Polymorphisms of Graphs

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For reflexive graphs

If H is an interval graph, then H admits a min ordering and a conservative majority

Existence of Polymorphisms of Graphs

For reflexive graphs

If H is an interval graph, then H admits a min ordering and a conservative majority

Otherwise H contains an asteroidal triple or a chordless cycle > 3 , and admits neither a min ordering nor a conservative majority

Brewster+Feder+H+Huang+MacGillivray 2007

Rafiey+H 2008

Existence of Polymorphisms of Graphs

For irreflexive graphs

If H is a bipartite graph whose complement is a circular arc graph, then H admits a min ordering and a conservative majority

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Feder+H+Huang 1999

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The list homomorphism problem for a graph H

If H has nice polymorphism, then the problem is polynomial

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If H has no nice polymorphism, then there is an obstruction, and the problem is NP-complete

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Obstruction detected in polynomial time



List Homomorphism Problem for Digraphs

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As before

If a list homomorphism problem for digraph H admits a conservative semi-lattice, majority, or Maltsev, then the list CSP for H is polynomial

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If each pair $u, v \in V(H)$ admits a conservative polymorphism f of H such that $f|_{\{u, v\}}$ is semi-lattice, majority, or Maltsev, then the problem is polynomial.

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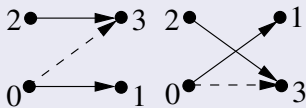
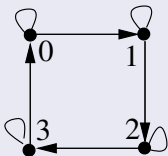
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Bulatov 2003

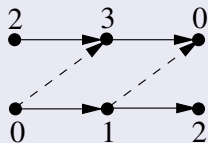
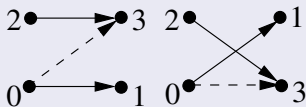
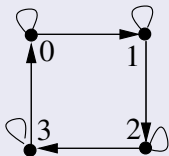
Existence of Polymorphisms of Digraphs

Reflexive 4-cycle does not have a min ordering



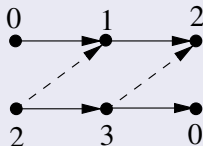
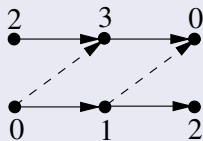
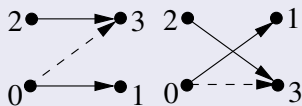
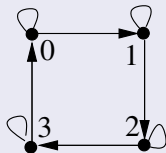
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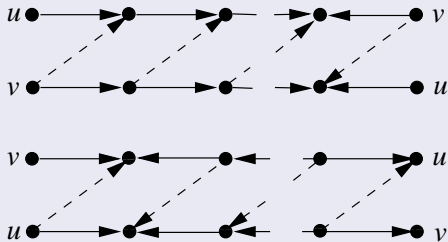
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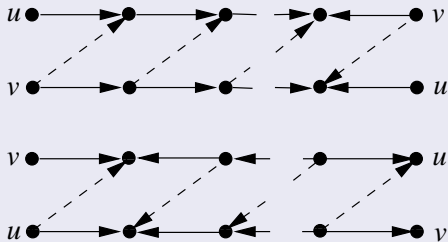
Existence of Polymorphisms of Digraphs

Invertible pair u, v



Existence of Polymorphisms of Digraphs

Invertible pair u, v



For reflexive digraphs

A min ordering exists if and only if H has no invertible pair

Feder+H+Huang+Rafiey 2009

For Reflexive Graphs

For reflexive graphs

H is an interval graph if and only if it has no invertible pair

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H is an interval graph if and only if it has no invertible pair

The following statements are equivalent for a reflexive graph H

- H has no asteroidal triple and no hole
- H has a consecutive clique enumeration
- H has no invertible pair

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Implies the theorems of Lekkerkerker-Boland and Fulkerson-Gross.



Existence of Polymorphisms of Digraphs

Majority H+Rafiey 2010

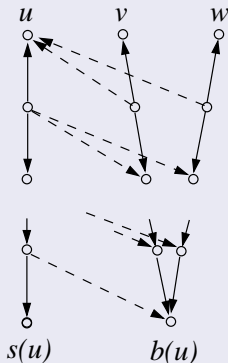
A digraph H admits a conservative majority if and only if it has no *permutable triple*

Existence of Polymorphisms of Digraphs

Majority H+Rafiey 2010

A digraph H admits a conservative majority if and only if it has no *permutable triple*

u is not the result of u, v, w

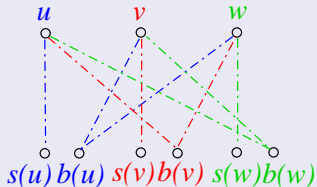


Existence of Polymorphisms of Digraphs

Majority H+Rafiey 2010

A digraph H admits a conservative majority if and only if it has no *permutable triple*

Permutable triple (in each permutation of u, v, w)



Existence of Polymorphisms of Digraphs

Majority

A digraph H admits a conservative majority **if** it has no permutable triple

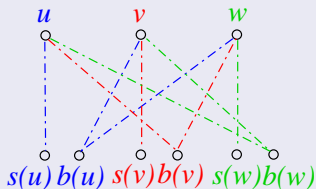
Existence of Polymorphisms of Digraphs

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Proof

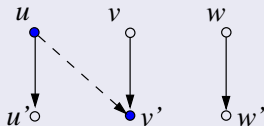
$g(u, v, w)$ is the first x in u, v, w without walks to $s(x), b(x)$



Existence of Polymorphisms of Digraphs

Proof

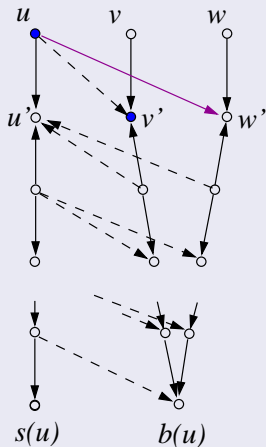
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Existence of Polymorphisms of Digraphs

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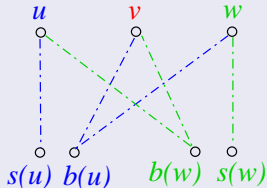


Existence of Polymorphisms of Digraphs

Maltsev H+Rafiey 2010

A digraph H admits a conservative Maltsev if and only if it has no end triple

End triple





An Auxiliary Digraph

Pair Digraph H^+

- Vertices: Pairs (u, v) with $u \neq v$
- Edges: From (u, v) to (u', v') if
 - $uu' \in E(H), vv' \in E(H), uv' \notin E(H)$
 - $u'u \in E(H), v'v \in E(H), u'v \notin E(H)$

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Skew-symmetry

$$(u, v) \rightarrow (u', v') \iff (v', u') \rightarrow (v, u)$$

An Auxiliary Digraph

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Skew-symmetry

$$(u, v) \rightarrow (u', v') \iff (v', u') \rightarrow (v, u)$$

Each strong component C of H^+ has a *coupled* strong component C' consisting of all (b, a) with $(a, b) \in C$

All conditions expressible in terms of H^+

The Pair Digraph

All conditions expressible in terms of H^+ , e.g.,

Invertible pair

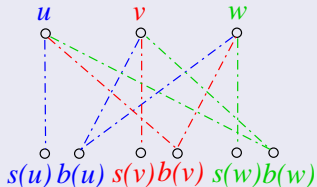
H contains an invertible pair if and only if H^+ has a self-coupled strong component.

Digraph Asteroidal Triples

DAT

A permutable triple u, v, w with each pair $(s(u), b(u)), (s(v), b(v)), (s(w), b(w))$ being invertible

Recall permutable triple



If H has a DAT

DATs make it hard

If H has a DAT then the list homomorphism problem for H is NP-complete

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$(s(u), b(u))$ does not admit semi-lattice, majority, or Maltsev
or
a direct proof, using the structure of a DAT

H+Rafiey 2010

If H has no DAT

DAT-free digraphs

If H has no DAT, then there exist polymorphisms f (binary) and g (ternary), such that

- $f|\{u, v\}$ is semi-lattice whenever (u, v) is not invertible
- $g|\{u, v\}$ is majority whenever (u, v) is invertible

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Corollary

If H is DAT-free, then the list homomorphism for H is polynomial

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Proof

Decide which $f(x, y) = x$ and which $f(x, y) = y$, so that for non-invertible pairs we have $f(x, y) = f(y, x)$

If $(x, y) \rightarrow (x', y')$ in H^+ , then $f(x, y) = x \implies f(x', y') = x'$

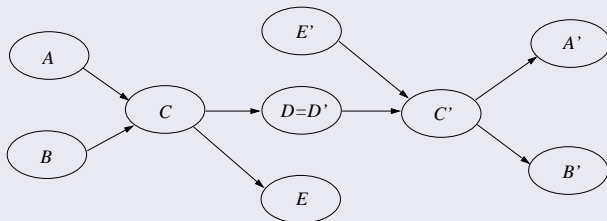
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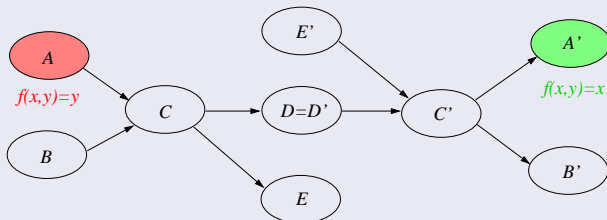
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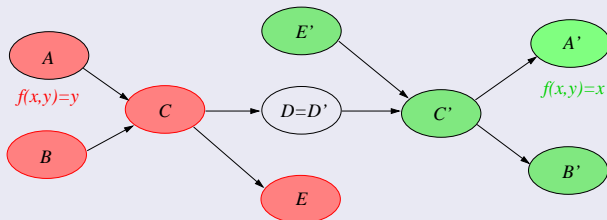
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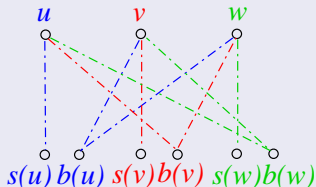
DAT-free digraphs

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Proof

$g(u, v, w)$ is the first x in u, v, w which occurs most frequently amongst $f(u, v), f(v, u), f(v, w), f(w, v), f(u, w), f(w, u)$ and has no walks to an invertible pair $s(x), b(x)$



Polynomial Dichotomy Classification for Digraphs

For a digraph H

If H is DAT-free, the list homomorphism problem for H is polynomial

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Testing for the existence of a DAT is polynomial

Simpler Polymorphism Characterization

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For a digraph H

If H is DAT-free, then for every pair $u, v \in V(H)$ there is a conservative polymorphism f of H such that $f|_{\{u, v\}}$ is semi-lattice or majority, and the list homomorphism problem for H is polynomial

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Width two suffices

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Valeriote 2009 + Barto-Kozik 2009



For Special Classes of Digraphs - Cycles

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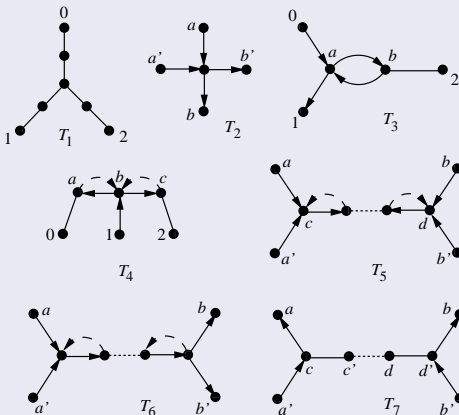
For an irreflexive oriented cycle H

- The homomorphism problem enjoys dichotomy Feder 2001
- A DAT-free H admits a conservative majority H+Rafiey 2010
- If H has a conservative majority polymorphism, then the list homomorphism problem for H is polynomial
Otherwise H contains a DAT and the problem is NP-complete

For Special Classes of Digraphs - Reflexive Trees

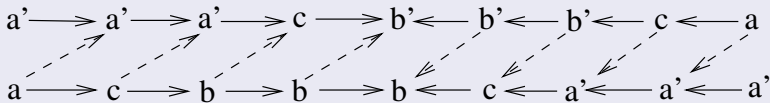
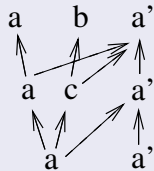
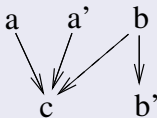
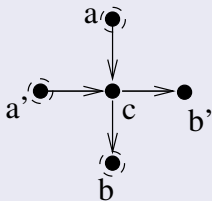
For Special Classes of Digraphs - Reflexive Trees

Minimal reflexive trees with DATs



For Special Classes of Digraphs - Reflexive Trees

A DAT in T_2



For a tree H

- A DAT-free tree H admits a min ordering H+Rafiey 2010

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- A DAT-free tree H admits a min ordering H+Rafiey 2010
- If H contains a DAT, the list homomorphism problem for H is NP-complete
Otherwise H has a min ordering and the problem for H is polynomial

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If H has a min ordering, then list homomorphism problem for H is polynomial, and otherwise it is NP-complete

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Carvalho+H+Rafiey 20??

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List homomorphism algorithm

Give a direct proof that if H is a DAT-free digraph, then 2-consistency checking solves the list homomorphism problem

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NP-complete for two binary relations

Bagan+Durand+Filiot+Gauwin 2010

