

# List colorings of planar graphs

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## EXTENDED ABSTRACT

Let  $G = (V, E)$  be a graph, let  $f : V(G) \rightarrow \mathbb{N}$ , and let  $k \geq 0$  be an integer. A *list-assignment*  $L$  of  $G$  is a function that assigns to each vertex  $v$  of  $G$  a set (list)  $L(v)$  of colors: usually each color is a positive integer. We say that  $L$  is an *f-assignment* if  $|L(v)| = f(v)$  for all  $v \in V$ , and a *k-assignment* if  $|L(v)| = k$  for all  $v \in V$ .

A *coloring* of  $G$  is a function  $\varphi$  that assigns a color to each vertex of  $G$  so that  $\varphi(v) \neq \varphi(w)$  whenever  $vw \in E$ . An *L-coloring* of  $G$  is a coloring  $\varphi$  of  $G$  such that  $\varphi(v) \in L(v)$  for all  $v \in V$ . If  $G$  admits an *L-coloring*, then  $G$  is *L-colorable*. The graph  $G$  is said to be *f-list-colorable* and  $f$  is called a *choice function* if  $G$  is *L-colorable* for every *f-assignment*  $L$  of  $G$ . When  $f(v) = k$  for all  $v \in V$ , the corresponding term becomes *k-list-colorable* or *k-choosable*. The *list-chromatic number* or *choice number*  $\chi_\ell(G)$  is the least number  $k$  such that  $G$  is *k-list-colorable*.

In 1979 Erdős, Rubin and Taylor [1] conjectured that all planar graphs are 5-list-colorable, but that not all planar graphs are 4-list-colorable. Both conjectures were proved in 1993.

**Theorem 1 ([4],[5])** *Every planar graph is 5-list colorable but there are planar graphs which are not 4-list colorable.*

Hence the situation seems to be clear for planar graphs. However we may ask for conditions which guarantee the 4-list colorability or even the 3-list colorability of planar graphs.

Many authors investigated the list colorability of planar graphs with forbidden cycle lengths. In this talk, we will discuss some aspects of this topic and give an overview on the recent state of research.

Another interesting direction of research deals with variations of the list lengths. We will consider some problems and results related to this topic. Finally, this investigations leads us to the concept of sum-list colorings. Instead of a fixed list length for all vertices we try to minimize the sum over all list lengths.

For a choice function  $f$  define  $size(f) = \sum_{v \in V} f(v)$ . The *sum choice number*  $\chi_{sc}(G)$  is the minimum of  $size(f)$  over all choice functions of  $G$ . The concept was introduced by Garth Isaak in 2002 [2]. An overview on the recent state of research is given in the PhD Thesis of Michelle Lastrina [3]

It is well-known that  $\chi_{sc}(G) \leq |V| + |E|$  for every graph  $G$ . To see this consider an arbitrary order  $v_1, \dots, v_n$  of the vertices of  $G$  and define  $f(v_i) := 1 +$  the number of vertices  $v_j$  adjacent to  $v_i$  with  $j < i$ . Obviously,  $\sum_{v \in V} f(v) = |V| + |E|$  and the graph is *L-colorable* by a greedy algorithm for every list assignment  $L$  with  $|L(v)| = f(v)$ . Therefore, a graph  $G$  is called *sc-greedy* if  $\chi_{sc}(G) = |V| + |E|$ .

**Corollary 2** *If  $G$  is a planar graph with  $n$  vertices then  $\chi_{sc}(G) \leq 4n - 6$ .*

**Corollary 3** *For every planar graph  $G$  there is a choice function  $f_G$  such that  $G$  is *L-list colorable* for every list assignment  $L$  with  $|L(v)| = f_G(v)$  and the average list length of such an assignment is*

$$\frac{\sum_{v \in V} |L(v)|}{|V|} \leq 4 - \frac{6}{n}.$$

In this talk we will discuss some problems and results concerning sum list colorings of planar graphs.

## References

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