

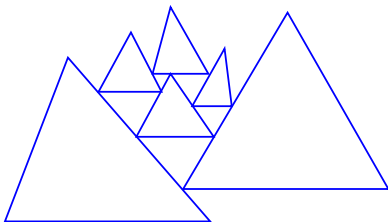
# Triangle Contact Systems for Planar Graphs.

Daniel Gonçalves  
LIRMM, Montpellier

BGW 2010

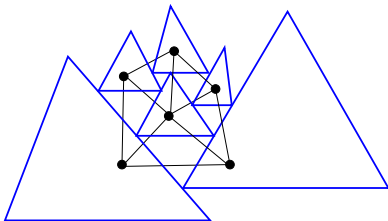
Theorem (de Fraysseix, Ossona de Mendez, Rosenstiehl '94)

Every planar graph admits a contact representation by triangles.

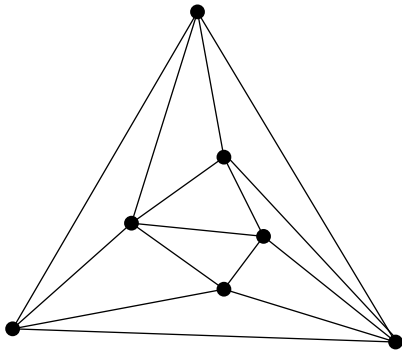


Theorem (de Fraysseix, Ossona de Mendez, Rosenstiehl '94)

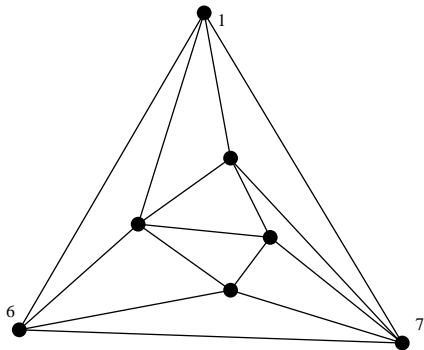
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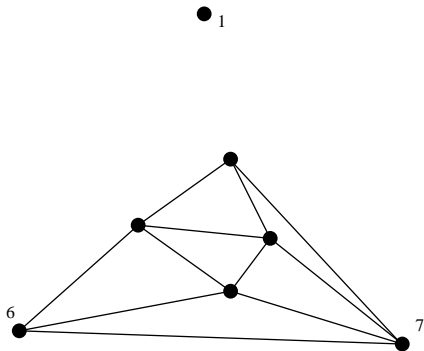
# Shelling Order in Triangulations



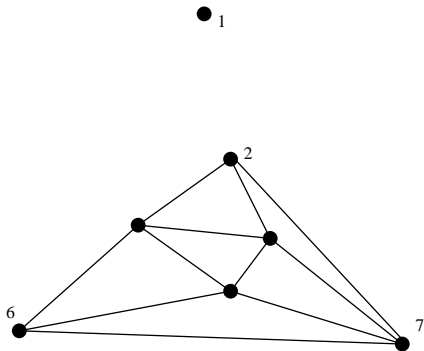
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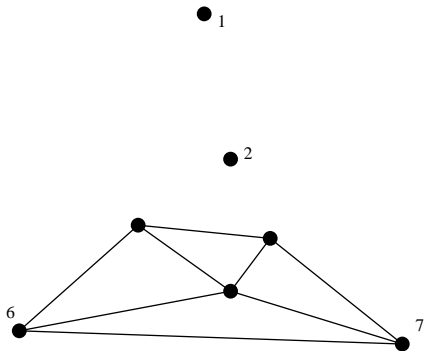
# Shelling Order in Triangulations



# Shelling Order in Triangulations

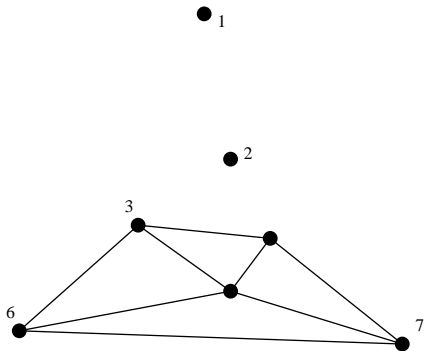


# Shelling Order in Triangulations

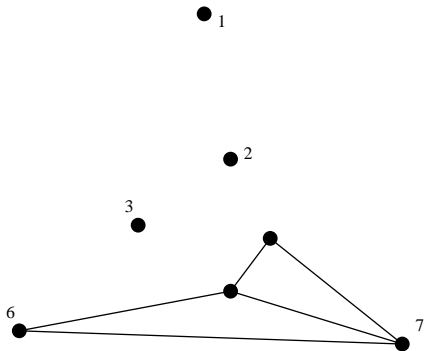




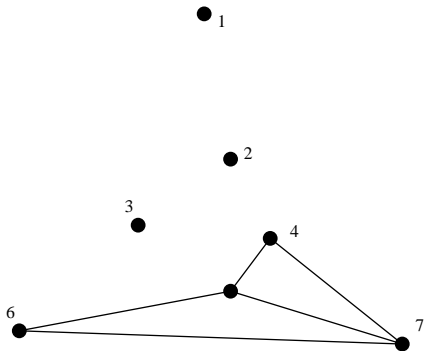
# Shelling Order in Triangulations



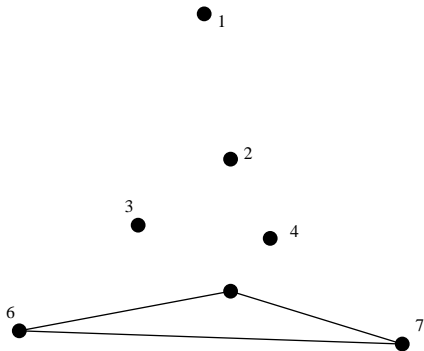
# Shelling Order in Triangulations



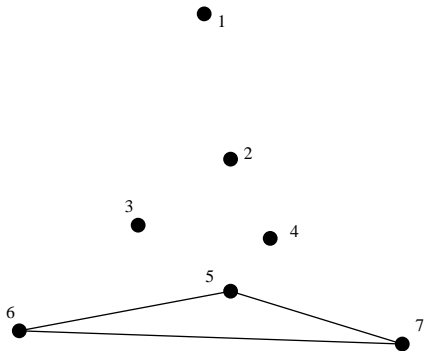
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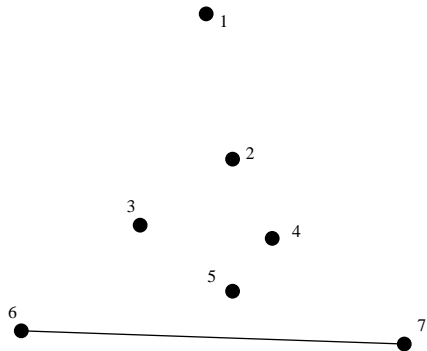
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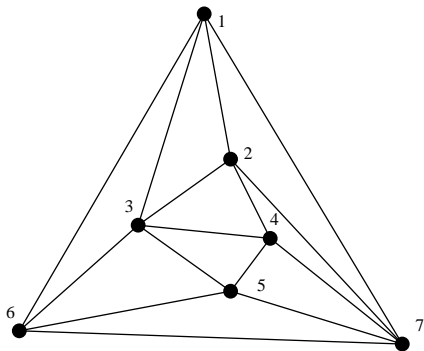
# Shelling Order in Triangulations



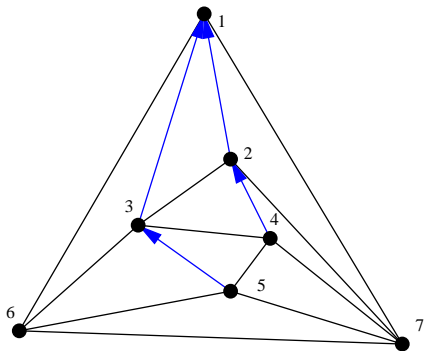
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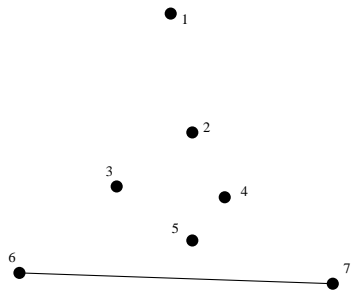


# Shelling Order in Triangulations

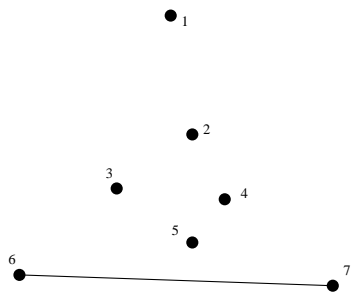




# Proof : Construction



# Proof : Construction



0 \_

1 \_

2 \_

3 \_

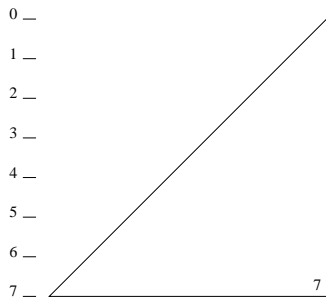
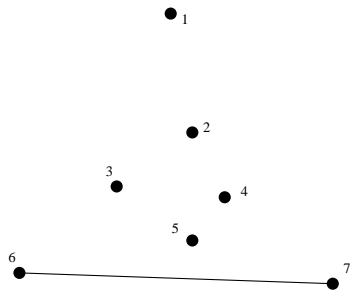
4 \_

5 \_

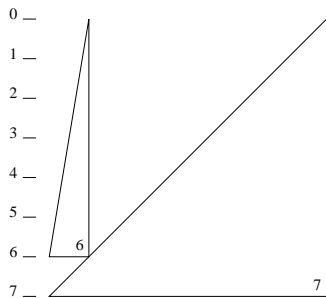
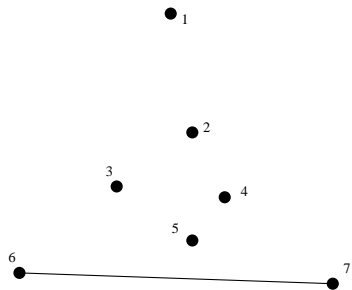
6 \_

7 \_

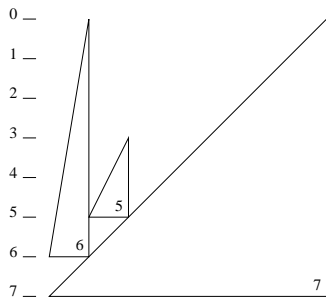
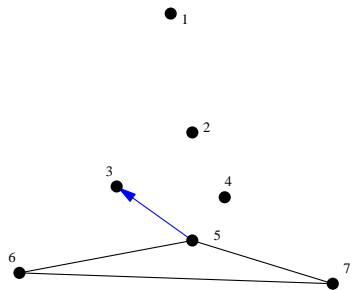
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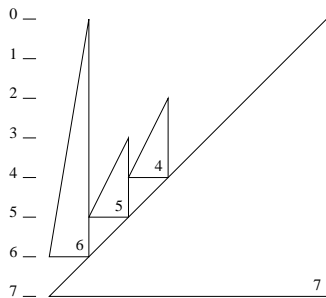
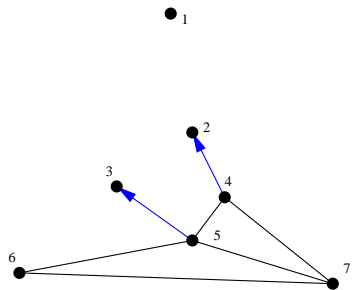
# Proof : Construction



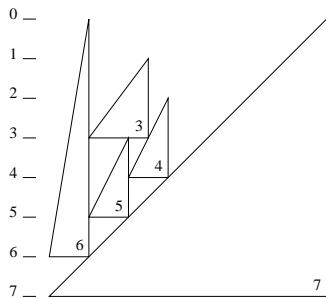
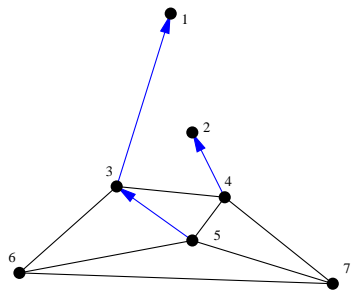
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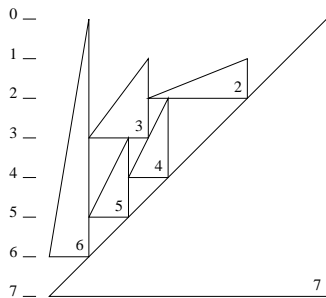
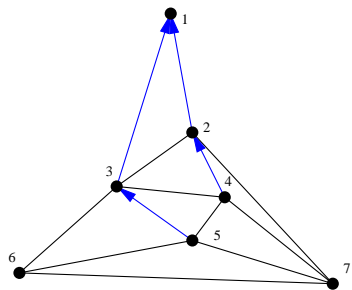
# Proof : Construction



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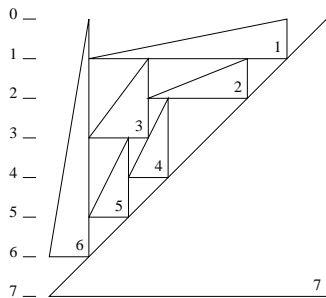
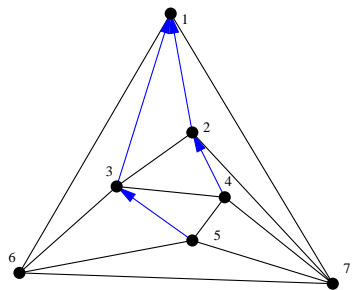


# Proof : Construction





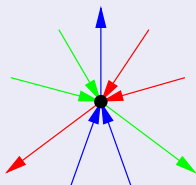
# Proof : Construction



# Correspondence with Schnyder woods

## Definition

A Schnyder wood of a triangulation  $T$  is an orientation of  $E(T)$  and a 3-edge coloring of  $T$  such that every inner vertex verifies :



## Correspondence with Schnyder woods

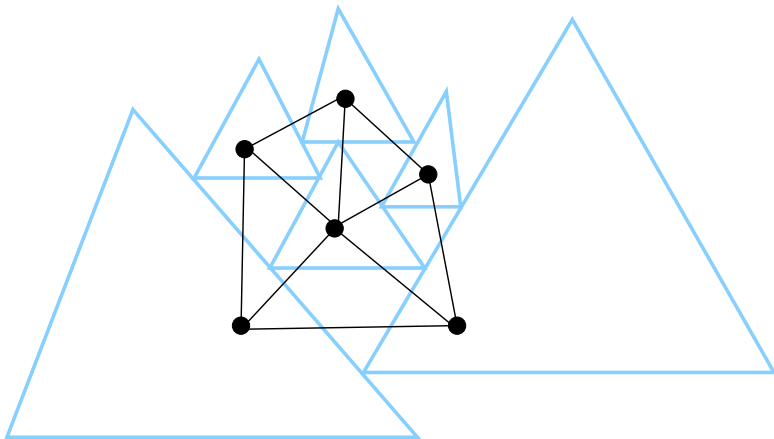
Theorem (de Fraysseix, Ossona de Mendez, Rosenstiehl '94)

Contact representations by triangles of a planar triangulation are in one-to-one correspondence with its Schnyder wood.

# Correspondence with Schnyder woods

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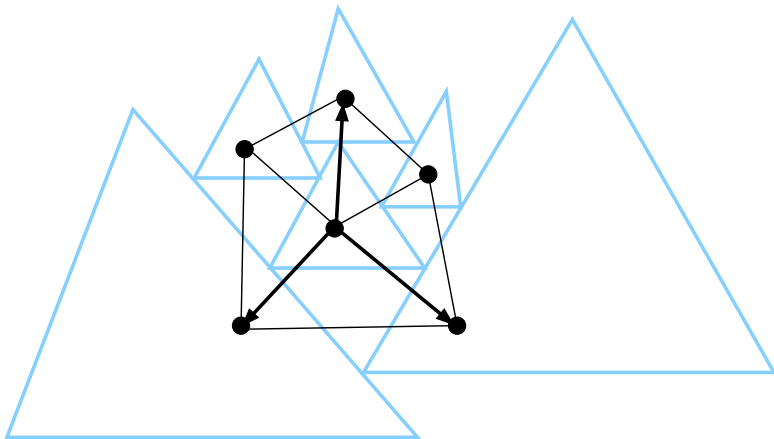
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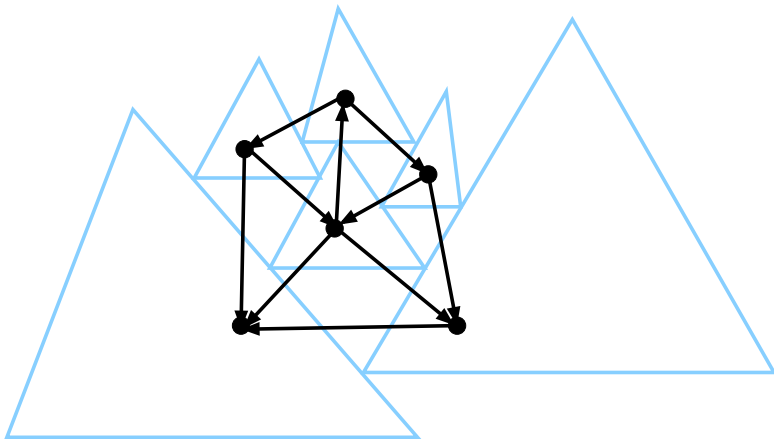
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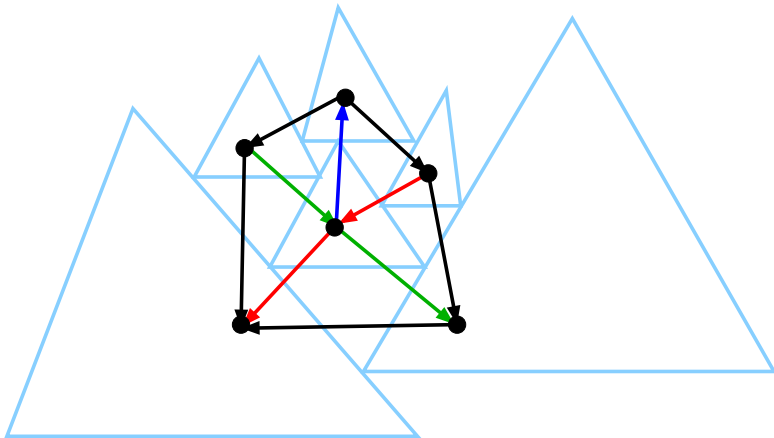
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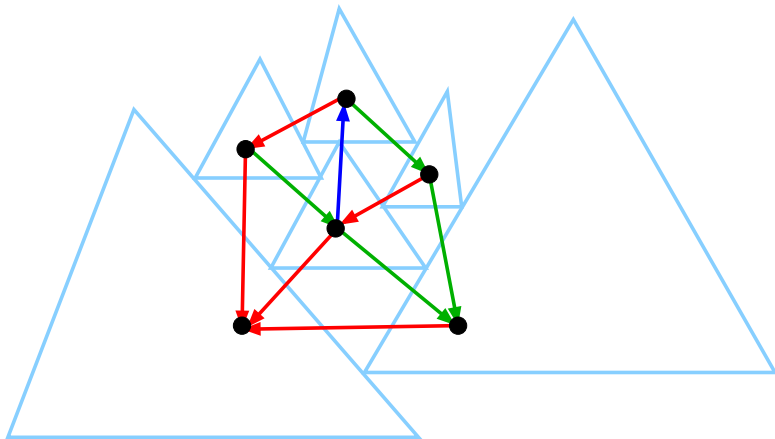
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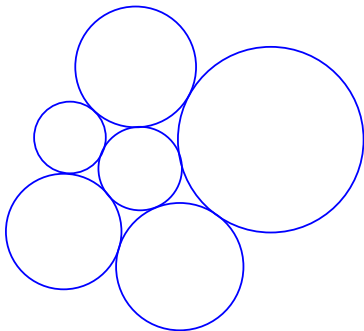




# Primal-Dual Representations

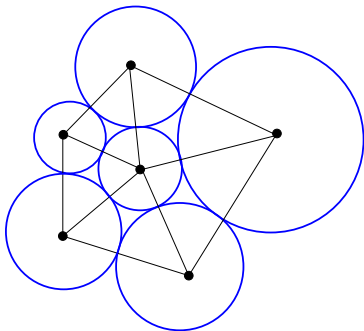
## Theorem (Koebe '36)

Every planar graph admits a contact representation by circles.



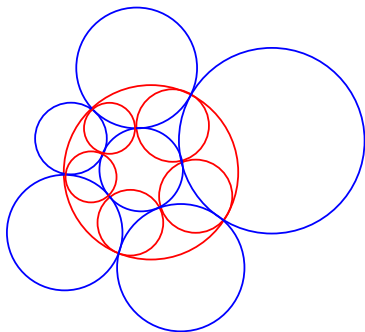
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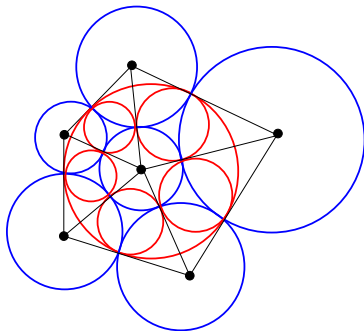
## Theorem (Andre'ev '70)

Every 3-connected planar map admits a primal-dual contact representation by circles.



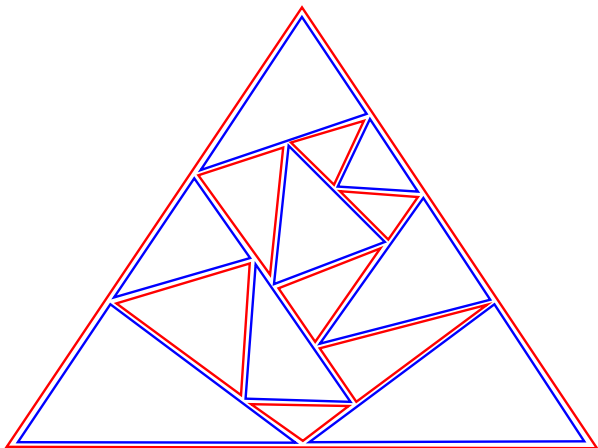
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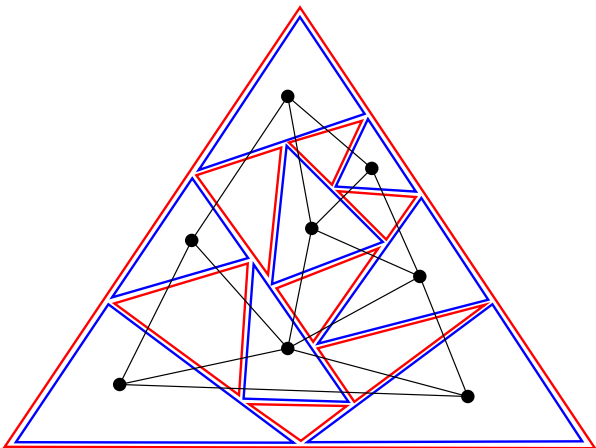
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Every 3-connected planar map admits a primal-dual contact representation by triangles.

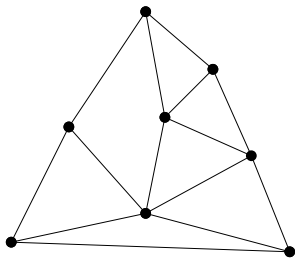


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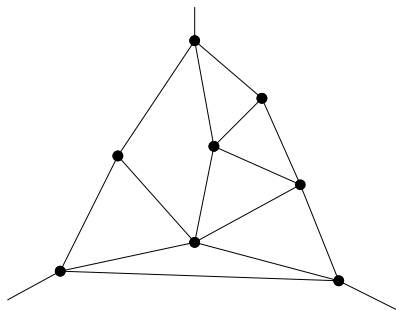


# Generalized Schnyder woods



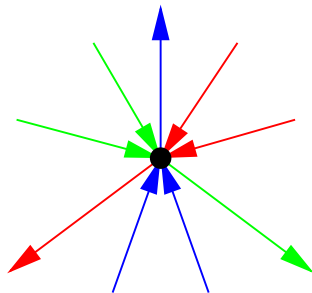


# Generalized Schnyder woods



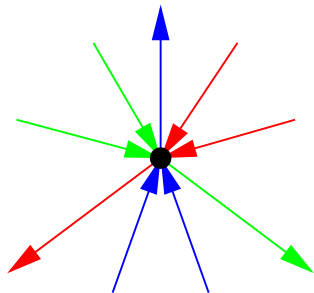
Add 3 edges on the outface

# Generalized Schnyder woods



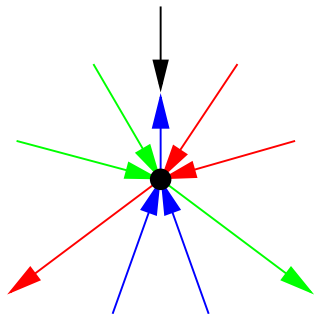
For inner vertices

# Generalized Schnyder woods



For EVERY vertex

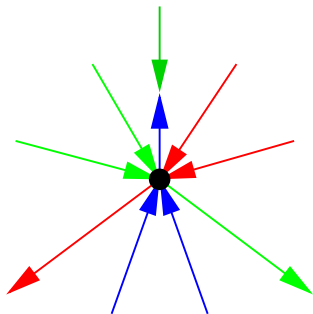
# Generalized Schnyder woods



For EVERY vertex

Allow BI-ORIENTED edges

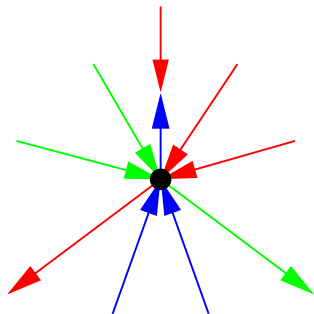
# Generalized Schnyder woods



For EVERY vertex

Allow BI-ORIENTED edges

# Generalized Schnyder woods



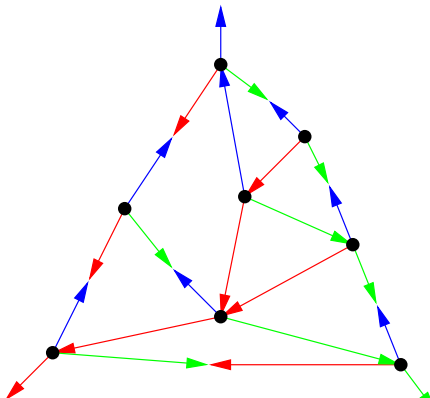
For EVERY vertex

Allow BI-ORIENTED edges

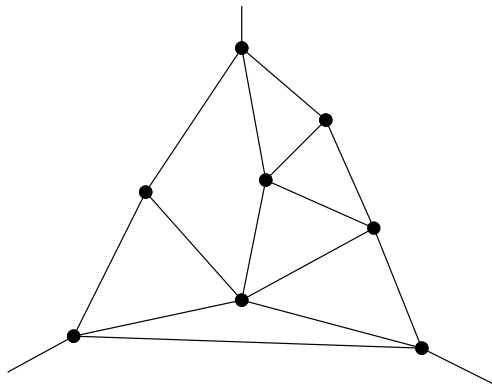
# Generalized Schnyder woods

## Theorem (Felsner '01)

Every 3-connected planar map admits a generalized Schnyder wood.

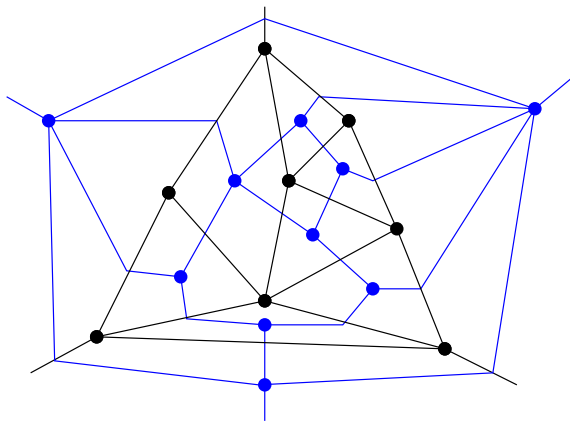


# Generalized Schnyder woods and Duality

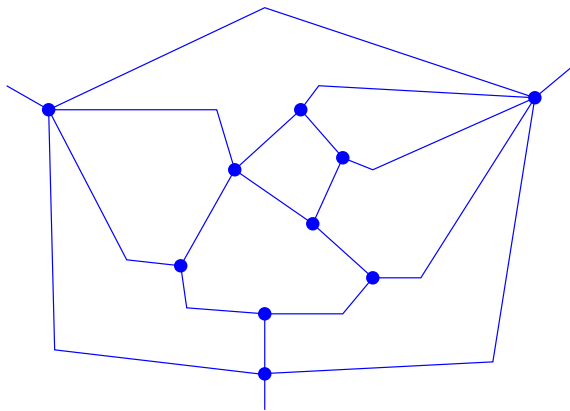




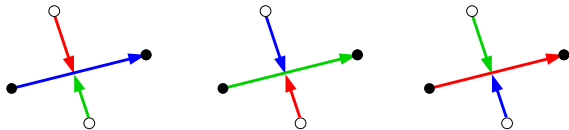
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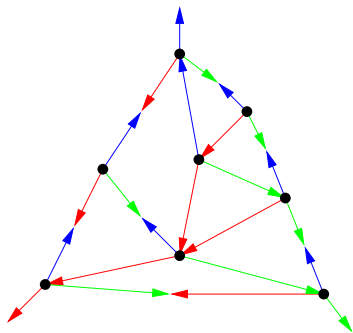
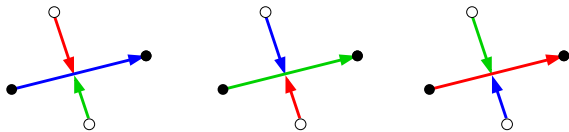
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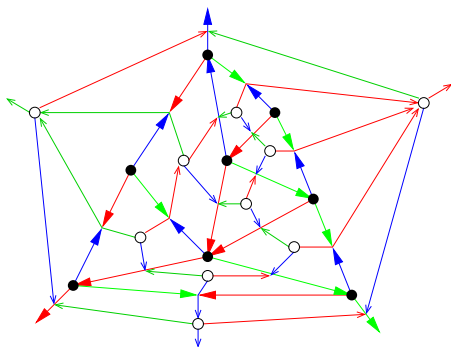
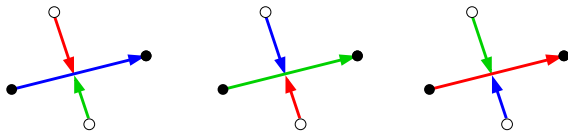
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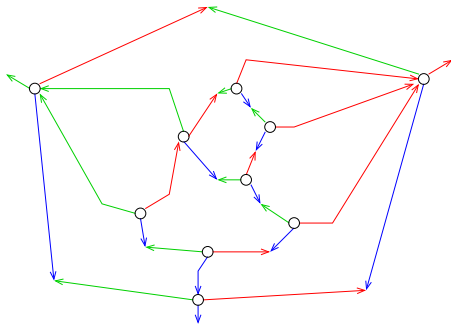
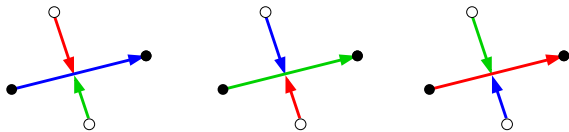
# Generalized Schnyder woods and Duality



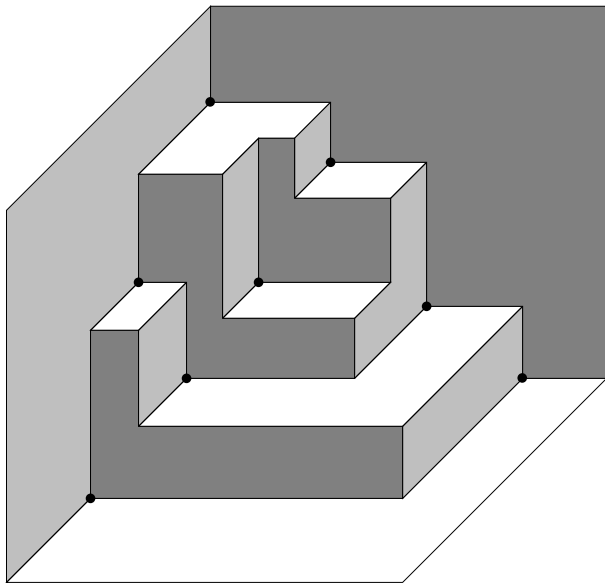
# Generalized Schnyder woods and Duality



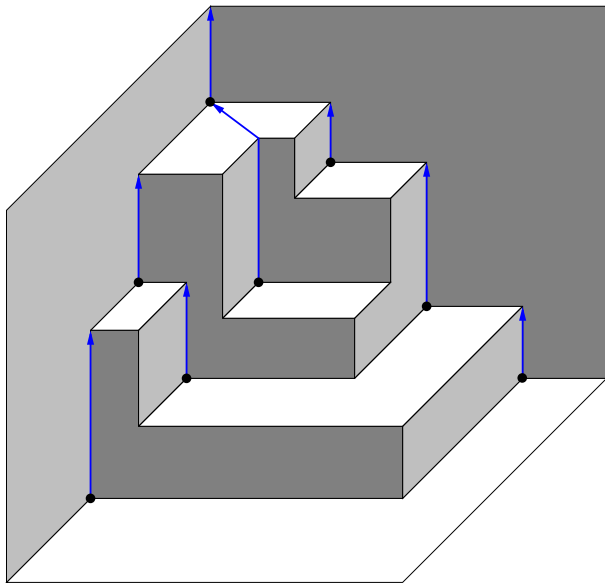
# Generalized Schnyder woods and Duality



# The Construction on an Orthogonal Surfaces

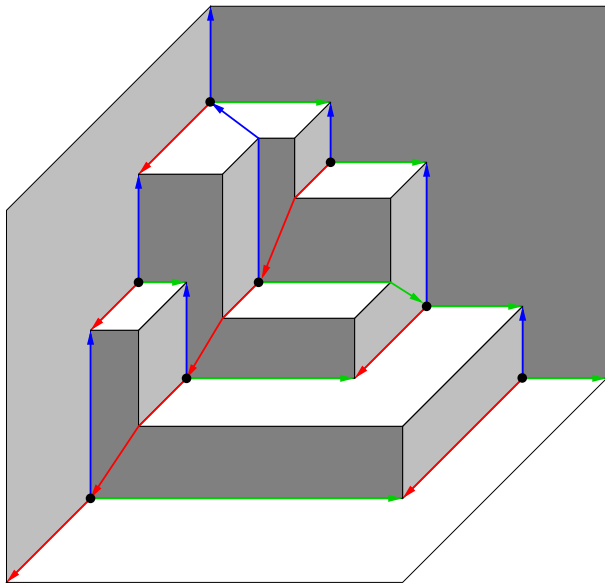


# The Construction on an Orthogonal Surfaces

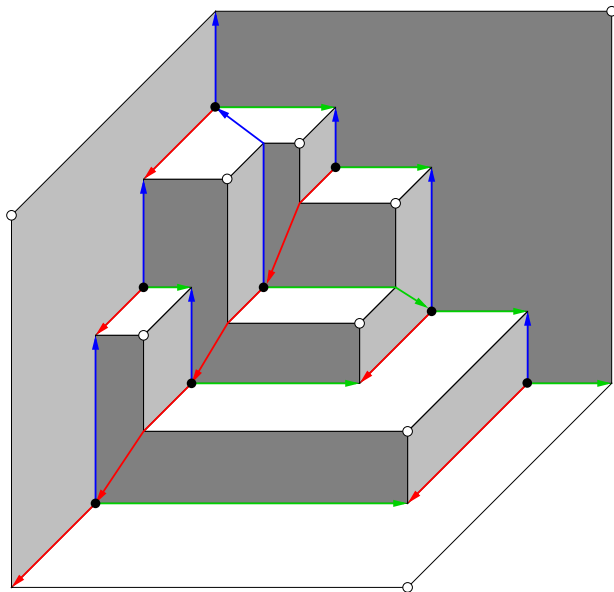




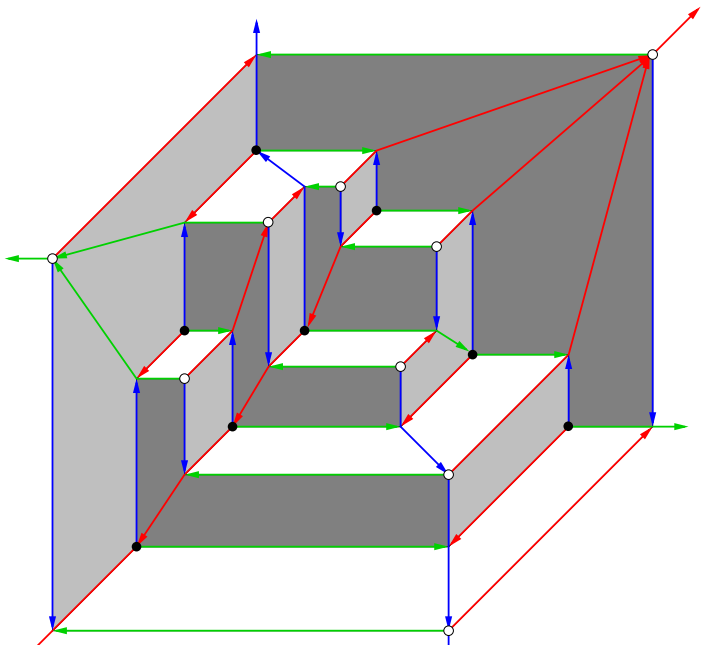
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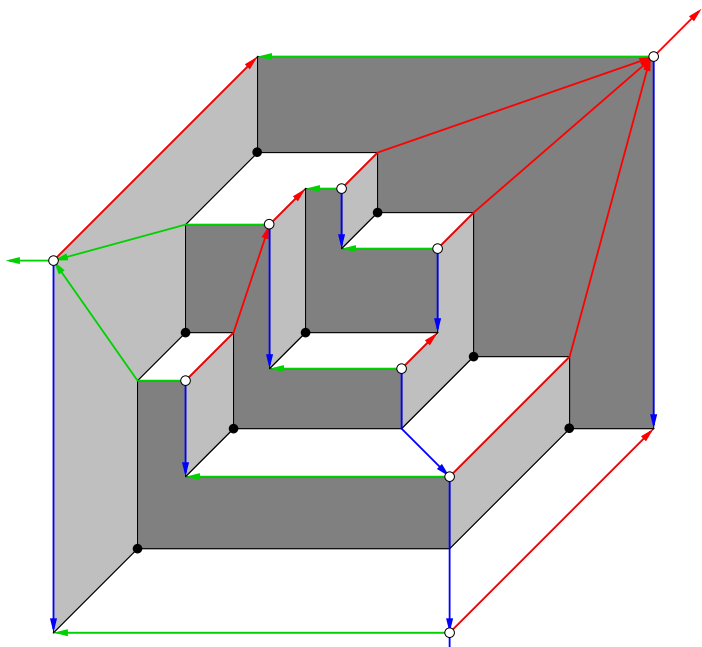
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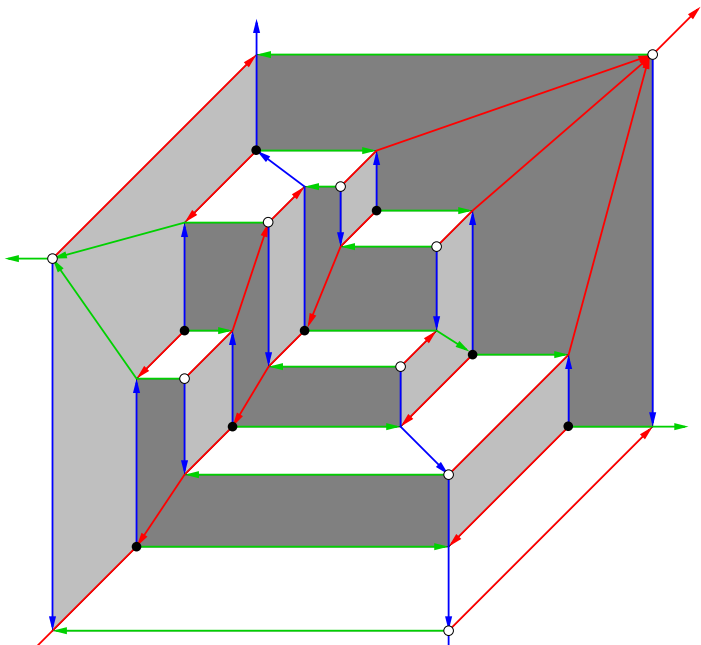
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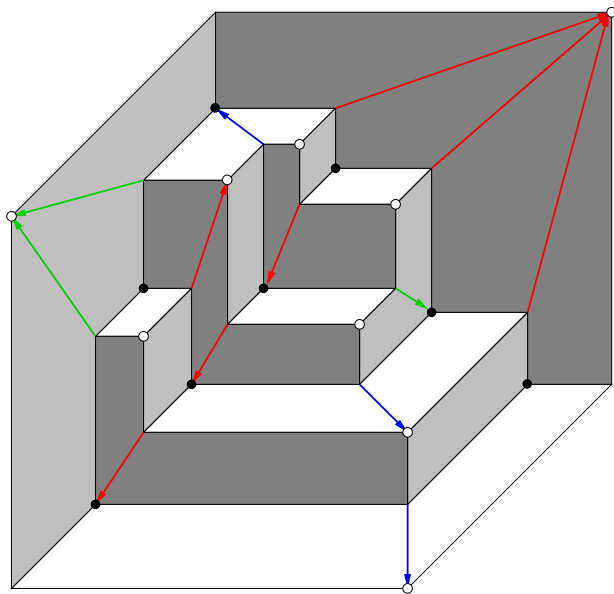
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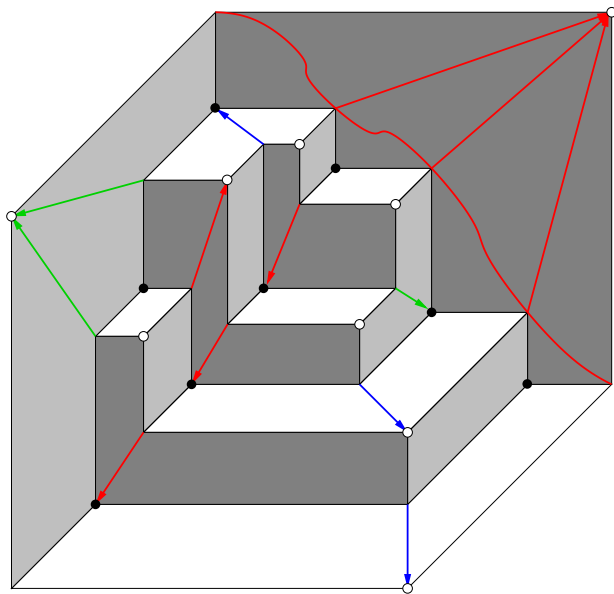
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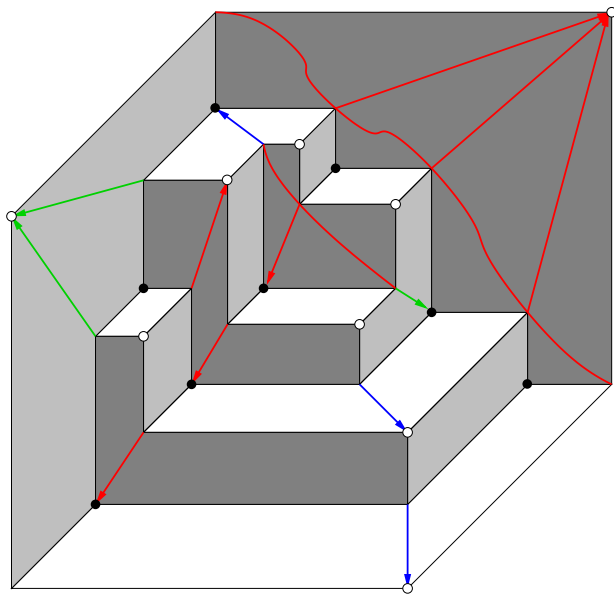
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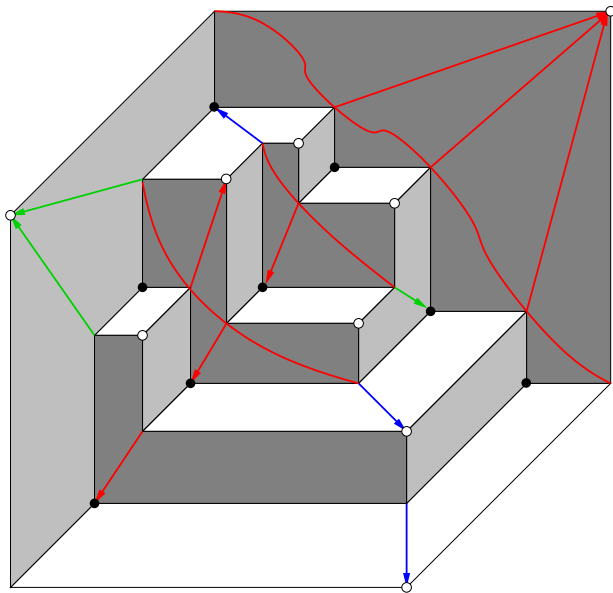


# The Construction on an Orthogonal Surfaces

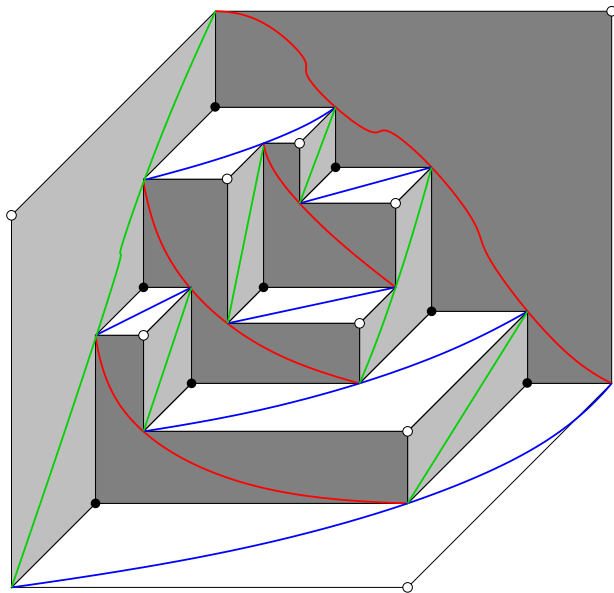




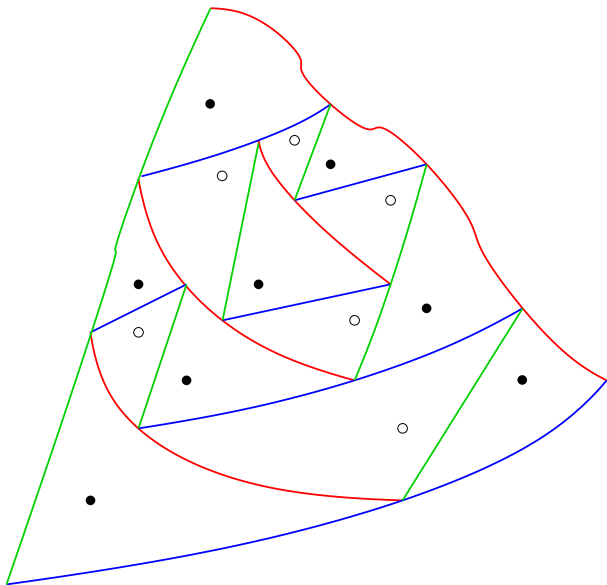
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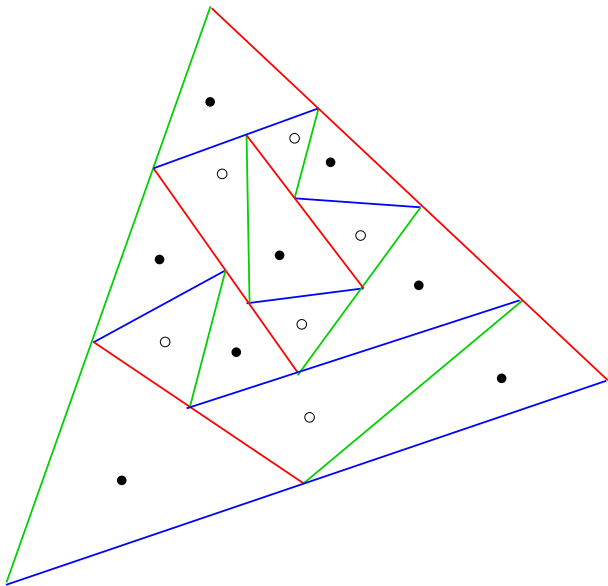
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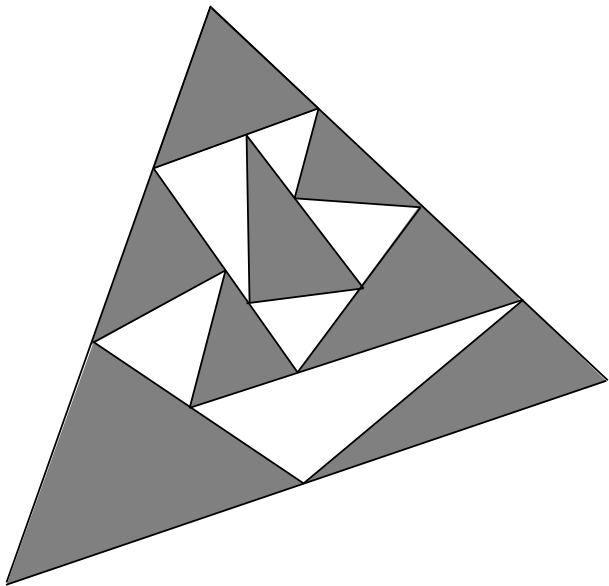
# The Construction on an Orthogonal Surfaces



# The Construction on an Orthogonal Surfaces



# The Construction on an Orthogonal Surfaces



# How to Stretch

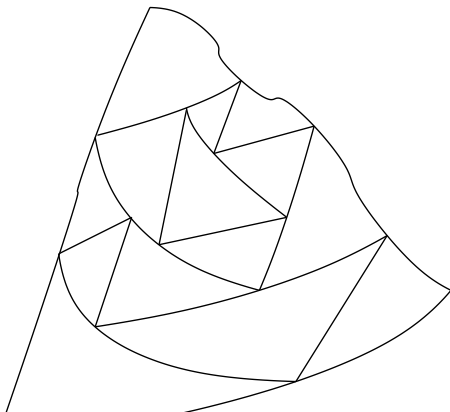
## Theorem (de Fraysseix, Ossona de Mendez '07)

A contact system of pseudo-segments (no loops or tangent arcs) is stretchable if and only if each subsystem of cardinality at least two has at least three extremal points.

# How to Stretch

## Theorem (de Fraysseix, Ossona de Mendez '07)

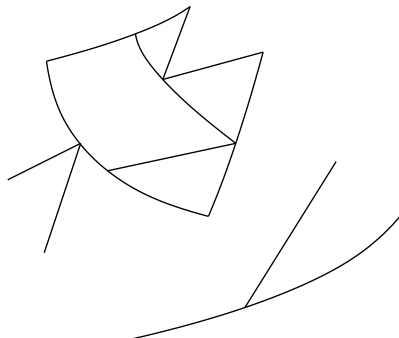
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# How to Stretch

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A contact system of pseudo-segments (no loops or tangent arcs) is stretchable if and only if each subsystem of cardinality at least two has at least three extremal points.

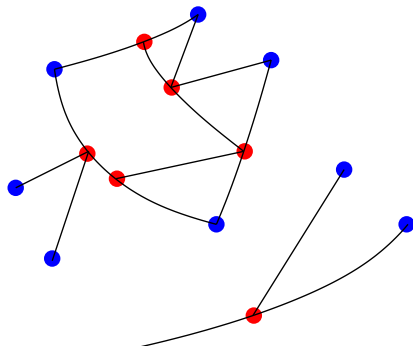




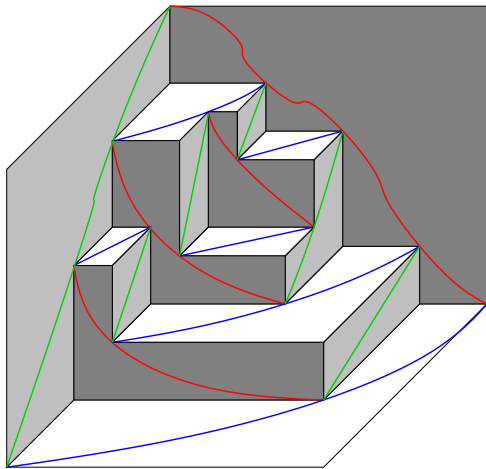
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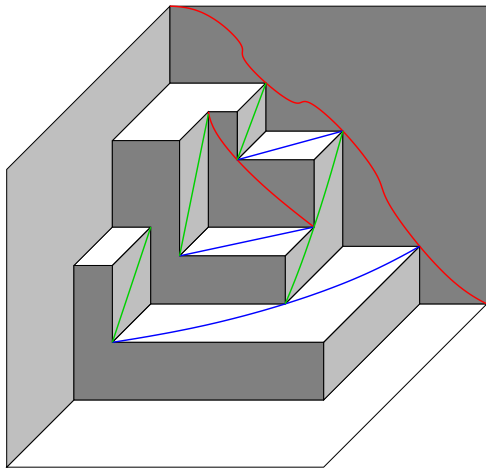
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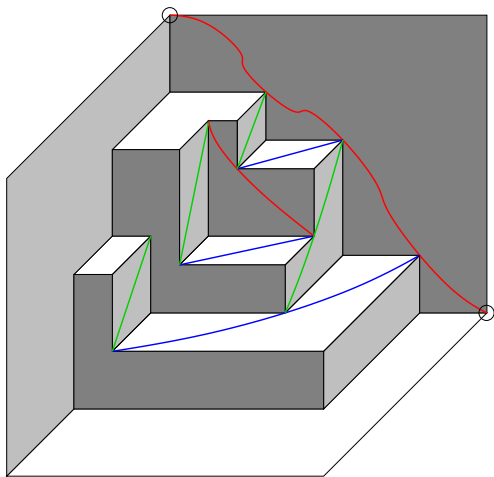
# How to Find Extremal Points ?



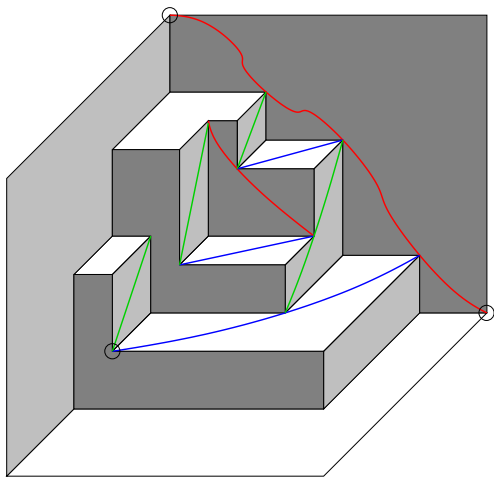
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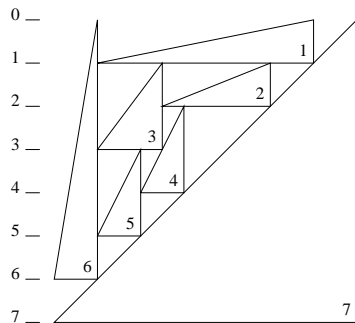


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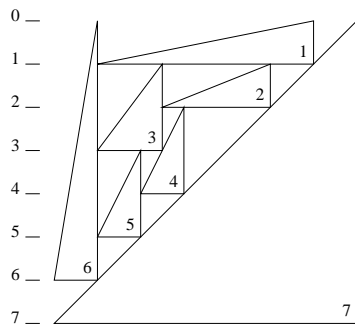
# Particular type of triangles ?

## ► Right triangles



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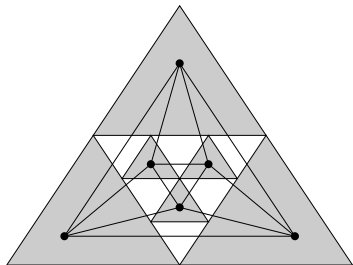
## Open Problem

Does every 3-connected planar map admits a primal-dual contact representation by right triangles?

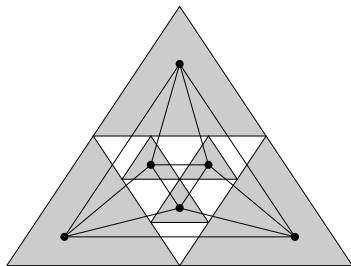
## Homothetic Triangles



# Contact of Homothetic Triangles



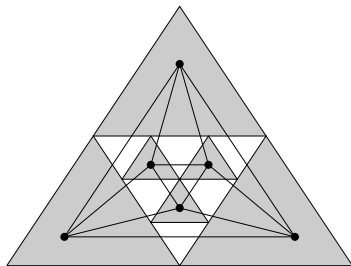
# Contact of Homothetic Triangles



## Conjecture (Kratochvil '07)

Every 4-connected planar triangulation admits a contact representation by homothetic triangles.

# Contact of Homothetic Triangles



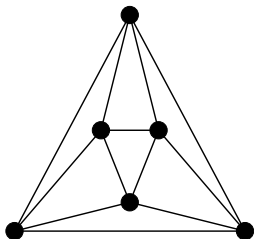
Theorem (by Schramm's Convex Packing Theorem)

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# Schramm's Convex Packing Theorem

## Theorem (Schramm '90)

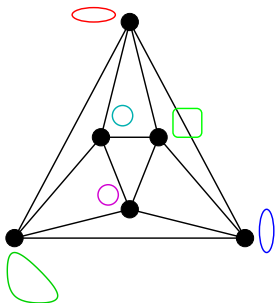
For any triangulation  $T$  and any convex and smooth bodies  $P_v$ , for  $v \in V(T)$ , there is a contact representation of  $T$



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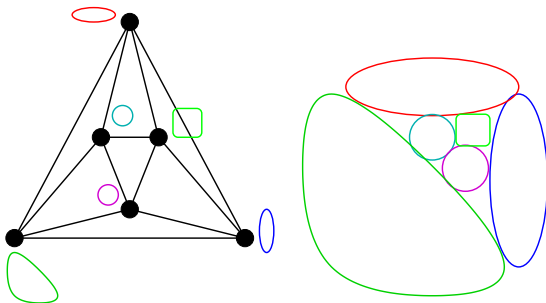
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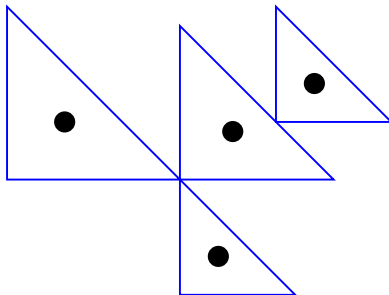
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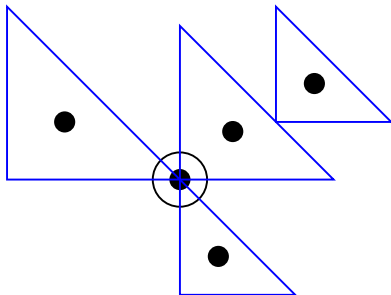




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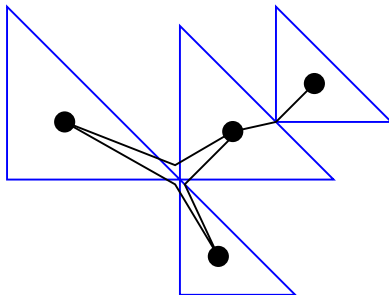
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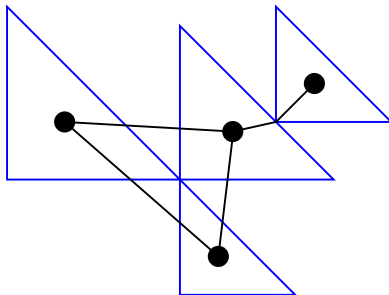
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# Intersection of Homothetic Triangles

## Theorem

Every planar graph is the intersection graph of homothetic triangles (without 3 intersecting triangles).

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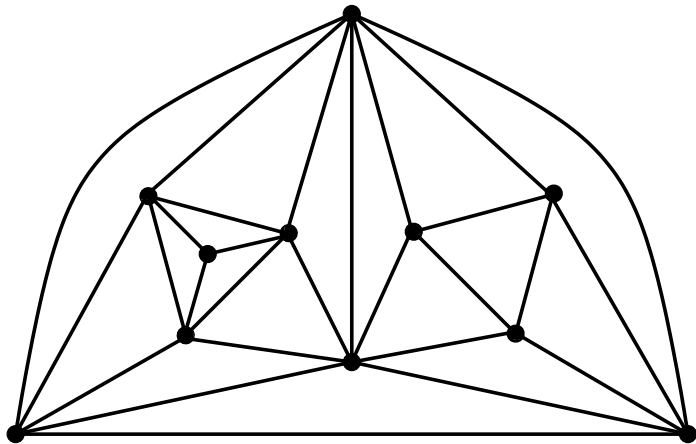
# Intersection of Homothetic Triangles

## Theorem

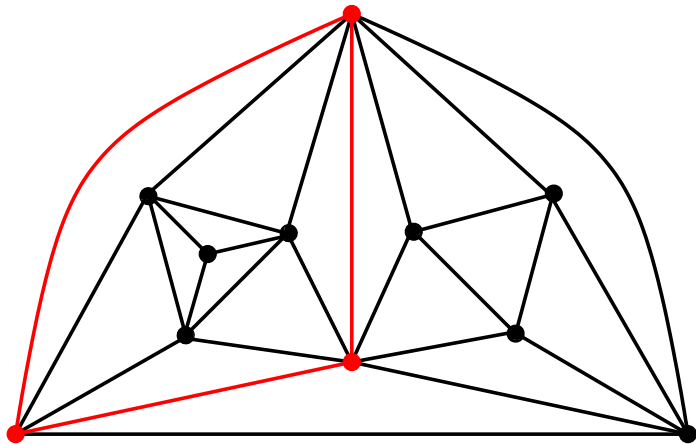
Every planar graph is the intersection graph of homothetic triangles (without 3 intersecting triangles).

- ▶ Restriction to triangulations.
- ▶ Work on the 4-connected components.

# Decomposition of Triangulations into 4CT

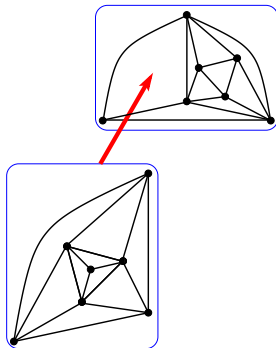


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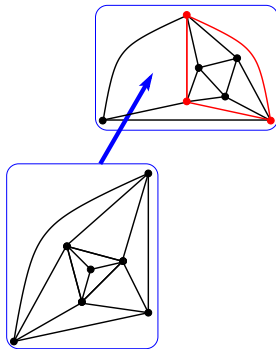




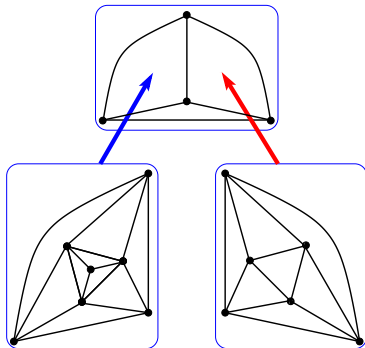
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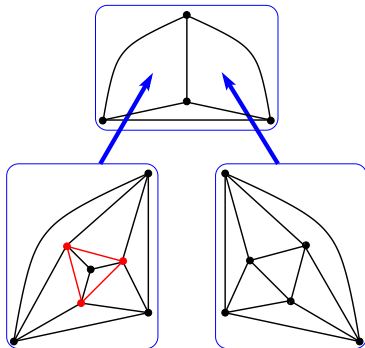
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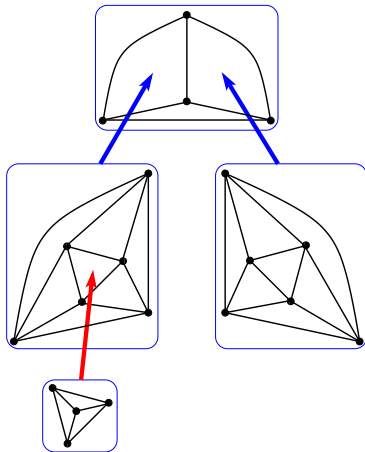
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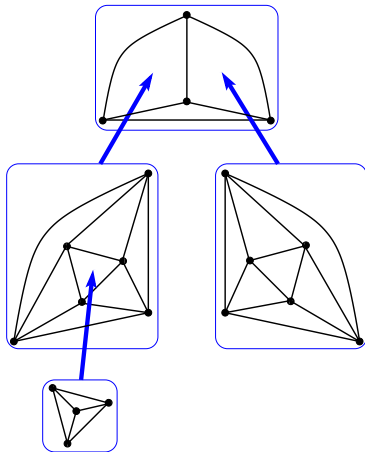
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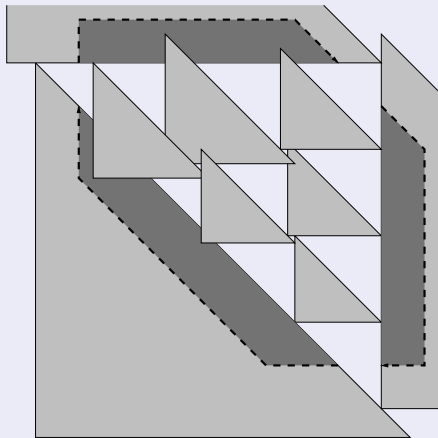


# 4-Connected Triangulations

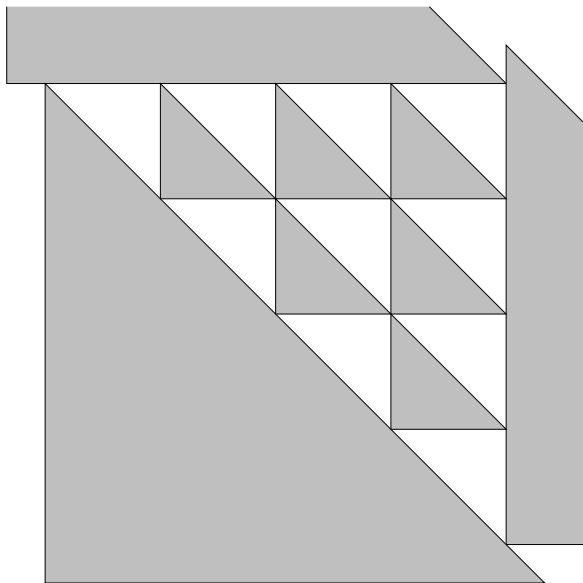
## Theorem

For every 4-connected triangulation  $T$  and every  $\epsilon > 0$  there is an intersection system of homothetic triangles like that :

- ▶ No 3 intersecting triangles

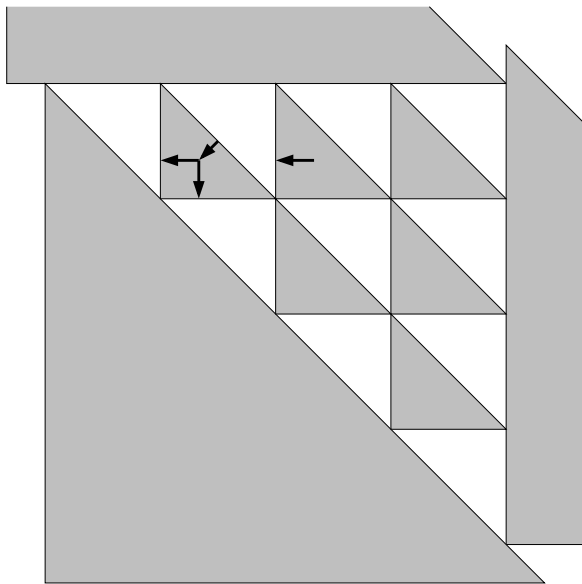


# Proof for 4-connected triangulations

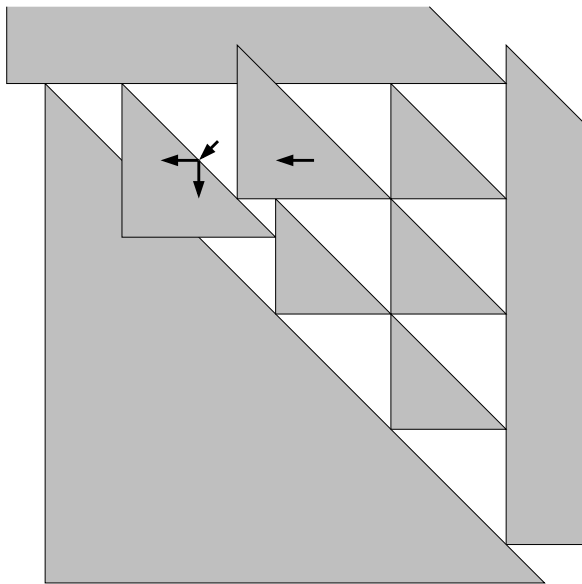




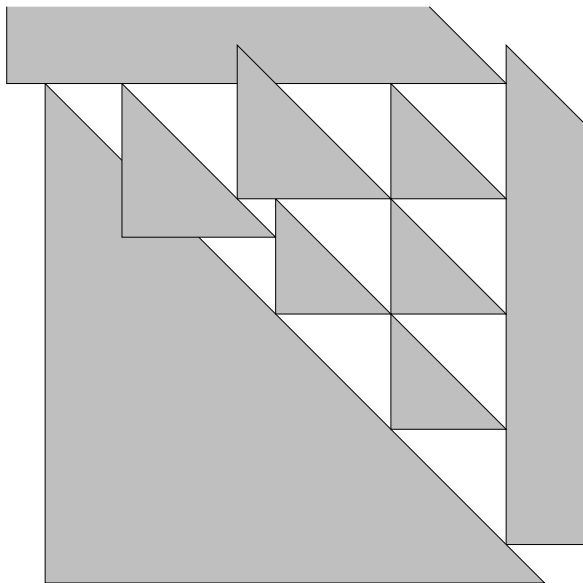
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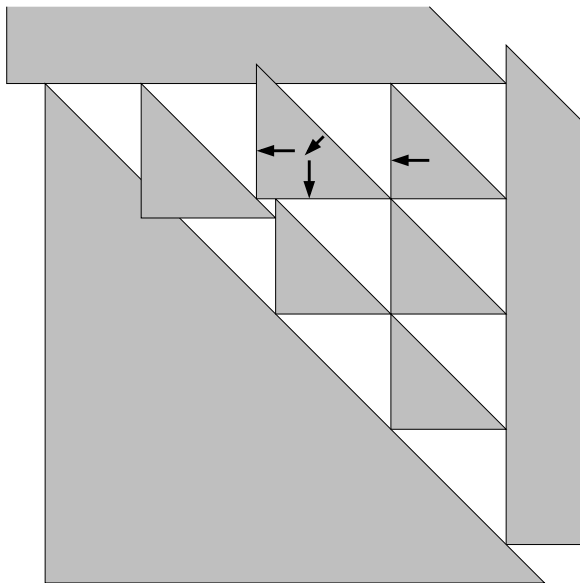
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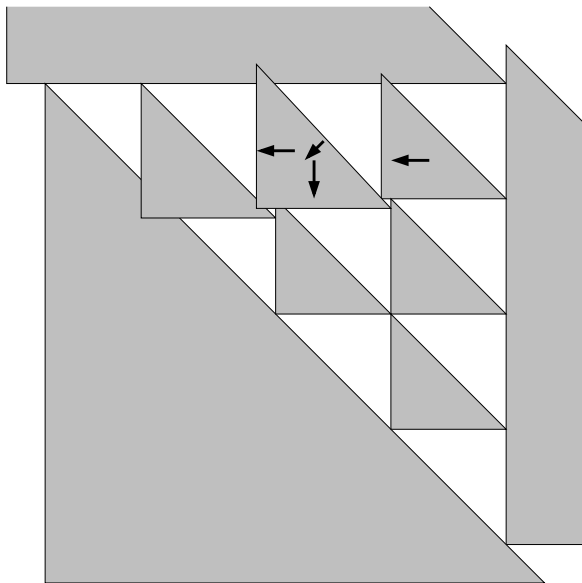
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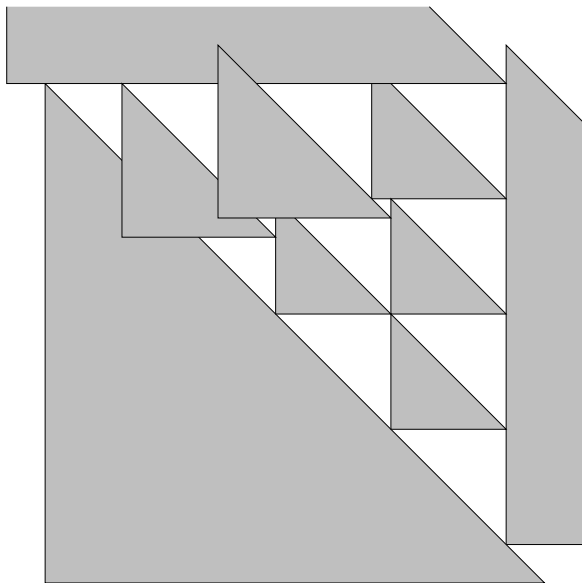
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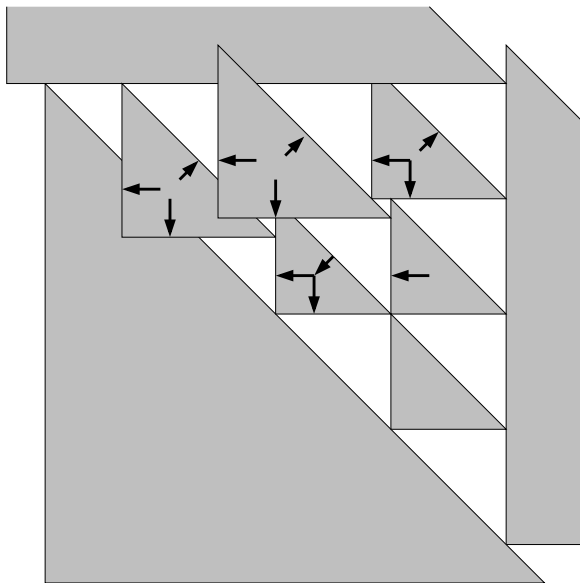
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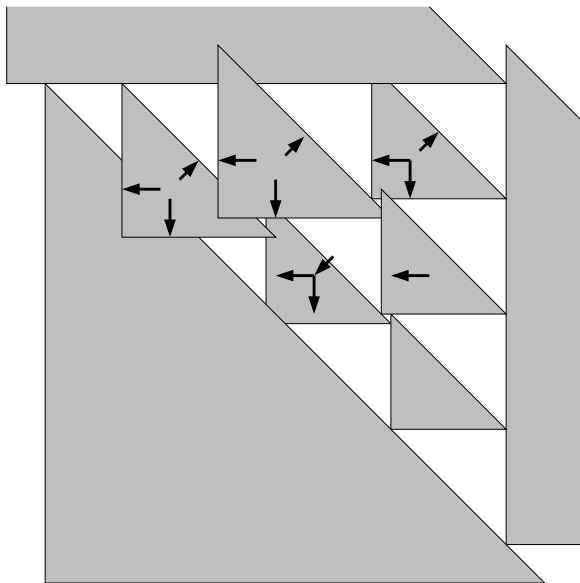
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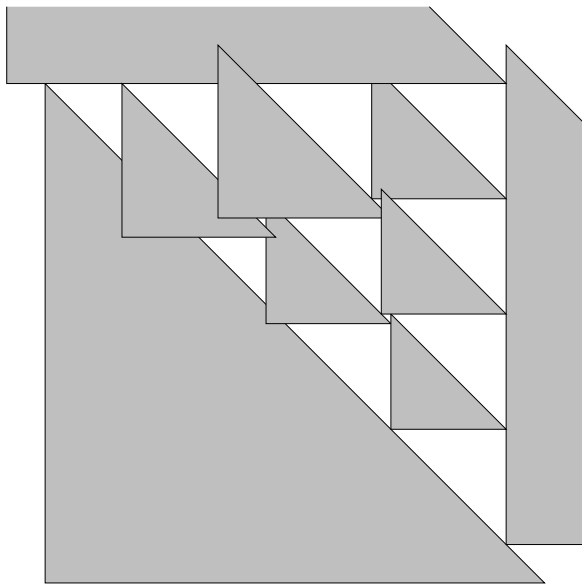


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Thank you !